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Analysis of equilibrium-oriented bidding strategies with inaccurate electricity market models

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ABSTRACT

In order to make competitive electricity markets effective, bidding generation companies (GENCOs) need to estimate market demand models according to information available to each of them. However, many stochastic factors (e.g. weather, demand side features) make it very hard for GENCOs to accurately capture the actual market demand in a model. Each GENCO might hold an estimated model deviating, from the real market model as well as from its peers'. Little work has been done in discussing the impacts of model deviations towards the design of GENCO's bidding strategies.

In this paper, the effects of model deviations upon the equilibrium-oriented bidding methods (EOBMs), more specifically conjectural variation (CV) based methods, are studied. We relax the strong assumptions that one uniform and accurate market demand model is employed by all GENCOs in the basic CV-based learning bidding algorithm (CVBA). In this work, the market demand model utilized for bidding by each GENCO is different from each other and from the actual market model as well. The impacts of such model deviations are analyzed from both theoretical and simulation perspective. Theoretical analyses point out that as a consequence of the model deviations it is possible that the basic CVBA algorithm will bring the bidding process into an unstable state. In order to eliminate the effects from inaccurate modeling, a CV-based learning bidding method with data filtering capabilities is proposed. Several sets of simulations have been done to test the impact of the model deviations. The simulation results confirm the theoretical analyses. The feasibility and effectiveness of the proposed bidding methods are also verified. The proposed algorithm can bring systems into stable state even when model deviations exist.

1. Introduction

In recent years, the power industry in many countries has been deregulated and has been opened up to competition. On one hand, thanks to enhanced competition, the social welfare and market efficiency are improved. On the other hand, traditional regulated generation companies (GENCOs) are facing more challenges to adapt to the daily operation of the new market environment. GENCOs are required to analyze the market themselves and to rationally conduct their strategic bidding. In such competitive markets, GENCOs have the freedom to exercise a certain degree of market power to maximize their profit. If GENCOs have complete information about the market, e.g. bidding strategy and marginal cost of other GENCOs', consumer response models, transmission network conditions etc., it would be possible for GENCOs to develop the optimal bidding decision accordingly. However, in reality many of those data – competitors' bidding strategies, for instance – might not be available for an individual GENCO. What a GENCO can have is publicly available information, such as historical demand data, market clearing prices, etc. In most cases, each GENCO should develop its bidding strategy with incomplete information.

In order to deal with this incomplete information, many proposals have been made with the intention of revealing the system by modeling the other market players' behaviors together with publicly available information.

One approach is to perceive the bidding curves submitted by the competitors as random values. In [1], the bidding curve coefficients submitted by rival suppliers are assumed to follow a joint normal distribution. The problem, originally with two decision variables, is reduced to one with only one decision variable. In [2], with a similar assumption, a fuzzy adaptive particle swarm optimization algorithm is proposed to solve the optimal bidding problem. In [3], an incomplete game has been transformed into a complete game by assuming the form of the probability distribution of competitors' behavior. The primary limitations of the approaches mentioned above are the impractical assumptions that the bidding behaviors of competitors strictly follow a certain

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distribution pattern. Moreover they neglect interactions between GENCOs despite the fact that in practice the actions taken by one GENCO will influence the others' behaviors and vice versa.

In other research, the use of multi-agent techniques is proposed to design the bidding strategy of a GENCO [4–7]. In those approaches, in order to capture the inter-influence between market players, an agent normally creates a state-action matrix to record all possible discrete states observed [4]. Then, reinforcement learning [5,7] or Erev-Roth algorithms [6] are used to derive the optimal bidding strategies for GENCOs. However, those approaches suffer scalability problems, as the dimensions of the action metric increase with the number of players in the system.

Market equilibrium models [8–15] have been introduced, as different interesting and promising category of approaches to model interactions among GENCOs in actual electricity markets. While an equilibrium point (also known as Nash Equilibrium [16]) is reached, no GENCO could increase its profit by unilaterally changing its behavior, e.g. its output. In many equilibrium-oriented models, conjectural variation (CV) methods are used to model the interactions among market players [17,18]. The essentials of conjectural variation value are to capture the behavioral response of competitors to the action change by the observed GENCO in the market. It has been well accepted that CV enables a more powerful representation of GENCO bidding behaviors and is capable of modeling various degrees of market competitions, ranging from perfect competition (CV = -1), Cournot game (CV = 0), to collusion (CV = 1) and other variants [18,19].

CV methods provide a quantified measure to analyze the bidding behavior of GENCOs. A duopoly market has been analyzed for a pool spot market [18], where CV is used to model/estimate forward market behavior. Ref. [10] analyzes the conjectural variation based equilibrium market in a competitive environment. Besides, Ref. [20] developed an empirical method within the CV method framework to evaluate and analyze the market behavior of GENCOs in a real world market. Ref. [21] researches on the outcome of an oligopolistic competition market when considering two types of linear conjecture functions as input to a market equilibrium model. The application of CV methods is not only limited to the power exchange market. Ref. [11,21–23] extend similar ideas with so called conjectured supply functions to model imperfect competitions on the electricity transmission market. However, those approaches focus on behavioral analysis with static and pre-assigned CV values. In fact, due to the complexity and dynamicity of markets, CV values are normally very hard to acquire and might vary with variation of GENCO's market behavior. For an actual electricity market, the pre-defined conjectures are normally inconsistent with the actual response of other GENCOs and need to be updated accordingly.

Two main categories of approaches to estimate CV values according to publicly available historical information, namely explicit fitting and implicit fitting, are employed in [1,11,24,25]. In an implicit fitting procedure a closed-form which employs historical available market data has been developed for energy and transmission price response [25]. However, those CV values only reflect system historical status and can only be used to analyze the static market behaviors of GENCOs within a predefined market setting. Moreover, in day-ahead markets or repeated markets based on regular time intervals, as each GENCO aims to maximize profits, normally they have incentives to learn from bidding history and public market data and hence, they gradually evolve their bidding behavior. Those static approaches fail to answer what CV value set will be reached in a dynamic market with multiple GENCOs.

In order to research the dynamic interaction among strategic GENCOs, in [15], a CV-based learning method is proposed, based on which GENCOs evolve their bidding behavior in a spot market. It has been proved that the equilibrium reached during the learning process is a Nash Equilibrium. The approaches typically assume a commonly agreed market model, e.g. a common market price demand function. However, for a practical electricity market, no such function exists. Each GENCO has to analyze publicly available information and its own private information to build its own market model. As the market model is typically influenced by many stochastic factors, e.g. changes of demand curves and behaviors of generators, each generator uses its own way to interpret the market data to construct its own estimated market model, based on which market behaviors are predicted. The models held by individual GENCO may be inconsistent with the real market model by minor variations.

This paper studies the behaviors of typical CV-based learning strategies in a repeated market from a novel yet practical perspective: firstly, market models (i.e. demand function) estimated by individual GENCO might differ from the actual market models; secondly, the market models employed by the involved GENCOs are not uniform anymore. An accurate uniform market function serves an assumption in many works on equilibrium-oriented bidding methods [10,15,22,23,26]. Practically no such commonly-agreed uniform market model exists. Even with the same historical market data, different GENCOs would have different interpretations of the market. The error resulting from deviations between GENCO's own market evaluation and the real one propagate during the repeated bidding process. This paper analyzes the impact of such model deviation on the CV-based learning bidding strategy. Firstly, CV-based learning bidding methods with an accurate uniform estimated market model are described and analyzed. Then, with the assumption (i.e. an accurate uniform market model) relaxed, it is pointed out that the model deviations would bring the basic CV-based learning algorithm to unstable states in some cases, which is undesirable for electricity system operation from both physical perspective and economic perspective. The analyses are further verified by a set of simulations. Afterwards, a CV-based dynamic learning algorithm with data filtering technology is proposed in this paper. The aim of the proposed algorithm is to alleviate the influence of the model deviations and to make the bidding process stable. The simulation results validate the effectiveness of the proposed algorithm.

The paper is arranged as follows: Section 2 describes the profit optimization problem for a GENCO in a competitive market. Section 3 presents the basic CV-based bidding method and the market equilibrium model. Section 4 firstly analyzes the impact of the deviation of modeling on repeated bidding process and then develops a CV-based method with data filter. The simulation results are shown in Section 5. Section 6 concludes the paper.

2. Profit maximization for a GENCO

In a competitive market, a GENCO, as a main market participant, has to strategically schedule its generation q_i to maximize its profit. The scope of this paper is limited to the market where a few independent GENCOs service a given geographic region where the transmission capacity is large enough. So a uniform market price is considered and the effect of congestion is neglected. For GENCO i, the profit maximization problem can be formalized as follows:

$$\begin{aligned} &\max_{q_i} \quad \Pi_i = \pi \cdot q_i - C_i(q_i) \\ &s.t. \quad q_{i\min} \leqslant q_i \leqslant q_{i\max} \end{aligned} \tag{1}$$

where Π_i is the profit of GENCO i. π is the market price which is an exogenous value for all GENCOs. $q_{i\min}$ and $q_{i\max}$ are the minimal and maximal generation capacity for GENCO i. $C_i(q_i)$ is the production cost function of GENCO i, which normally takes the form of the following quadratic function:

$$C_i(q_i) = \frac{1}{2}c_i q_i^2 + b_i q_i + a_i \tag{2}$$

c_i, b_i and a_i are the coefficients of the generation cost curve for GENCO i. For this cost function to be well-behaved, $c_i > 0$, $b_i > 0$ and $a_i \geqslant 0$ are satisfied.

From (1) we can see that the profit for a GENCO is determined by both the market price and the bidding quantity. Moreover the market price is affected by the behavior of market participants, within the regulated price cap. In order to achieve maximum profit, it is reasonable for GENCOs to evolve their bidding strategy by analyzing the behavior of its competitors from the available historical bidding and market data. Learning is needed to track the practical dynamic interactions that exist in the market: such behavior pattern need to be recognized for future decision making.

In this paper, the CV value is used to represent interactions of market participants. Solving GENCO's profit maximization problems is developed via CV-based bidding method. Firstly, a basic CV-based bidding method with a uniform market demand model is described. Then a more practical scenario is analyzed, where market demand models held by all GENCOs are not uniform anymore and deviate from the actual market model. The impact of such model deviations on CV-based bidding strategies for GENCOs is analyzed. Finally a new CV-based bidding algorithm with data filtering is proposed to overcome the impact of model deviations.

3. Conjectural variation method and market equilibrium model

3.1. One time period conjectural variation bidding framework

Supposing there are N GENCOs in the market; they compete with each other to supply electricity. The total system demand D and the supply quantity of GENCO i q_i satisfy:

$$D = \sum_{i=1}^{N} q_i = q_i + \sum_{j=1(j\neq i)}^{N} q_j = q_i + q_{-i} \tag{3}$$

where $q_{-i} = \sum_{j=1(j\neq i)}^{N} q_j$ is the total power supplied by the pseudo-competitors of GENCO i.

The market price π is interrelated with the system demand D via the market inverse demand function:

$$\pi = f(D) = f(q_i + q_{-i}) \tag{4}$$

The derivative of the above function associated with any positive demand value is negative. This function actually reflects the market behavior in function of both demand and time. In order to make a balance of modeling accuracy and analytical complexity, the demand function is assumed to be time-independent [10,17,18,24]. The static linear decreasing demand function form is commonly used:

$$\pi = A - kD = A - k(q_i + q_{-i}) \tag{5}$$

where coefficient $A(>0)$ is the intercept and $k(>0)$ is the slope of the demand curve.

When GENCO i reaches its optimal bidding quantity, the derivative of the profit in (1) associated with the bidding quantity q_i equals zero, that is:

$$\frac{\partial \prod_i}{\partial q_i} = \frac{\partial \pi}{\partial q_i} \cdot q_i + \pi - \mathrm{MC}_i(q_i) = 0 \tag{6}$$

where $\mathrm{MC}_i(q_i) = \frac{\partial C_i(q_i)}{\partial q_i}$ is the marginal cost for GENCO i when the production quantity is q_i. When Eq. (2) is taken, $\mathrm{MC}_i(q_i) = c_i q_i + b_i$.

If the demand curve has the form (5), the derivative of the market price associated with q_i is:

$$\frac{\partial \pi}{\partial q_i} = -k \cdot \left(1 + \frac{\partial q_{-i}}{\partial q_i}\right) = -k \cdot (1 + \mathrm{CV}_i) \tag{7}$$

The derivative $\frac{\partial q_{-i}}{\partial q_i}$ is defined as the conjectural variation CV_i of GENCO i, which measures the influence of the strategy variation of GENCO i on the strategy change of other competitors' [19]. There are other equivalences to the form of the conjectural variation mentioned above. For instance, the derivative of the market price with respect to the competitors' strategy, i.e. $\frac{\partial \pi}{\partial q_{-i}}$, is also appearing in some research [9,21].

Substituting (6) with (5) and (7) and rearranging give the optimal bidding quantity for GENCO i associated with CV_i:

$$q_i = \frac{A - k \cdot q_{-i} - b_i}{k \cdot (2 + \mathrm{CV}_i) + c_i} \qquad s.t. \quad q_{i\min} \leqslant q_i \leqslant q_{i\max} \tag{8}$$

Eq. (8) demonstrates the dominant factors which could influence the bidding quantity for a GENCO under CV method. It shows that,

(1) If the accurate market demand function is known, i.e. if k and A are determined and shared by all involved GENCOs, then for GENCO i, its optimal bidding quantity can be inferred with the CV_i value and the pseudo-competitors' bidding quantity $q_{-\mathrm{i}}$. The limited information needed in the bidding decision process makes CV-based bidding methods appropriate for an actual incomplete information electricity market.
(2) For different GENCOs, even with similar characteristics, the different adopted CV values will result in different bidding strategies.
(3) Modifying the CV values could lead to an increase or decrease of a GENCO's profit.

In the framework presented in [8–11], the CV values are assumed to be static and would not change in the repeating bidding process. However, in a practical electrical power market, each GENCO will adapt its bidding strategy to a dynamic market for the purpose of profit maximizing. Thus the conjectures on other behavior are not constant. If the GENCOs want to win such game it is reasonable that they learn from historical bids and update their conjecture towards market situation and adjust their bidding strategy accordingly.

3.2. Market equilibrium model

Once the bidding of each GENCO is modeled with the integration of other competitor behaviors' evaluation, the market equilibrium model becomes a collection of each generator's equilibrium model together with market clearing models, as follows:

$$q^* = \begin{cases} q_1^* = \frac{A - k \cdot q_{-1} - b_1}{k \cdot (2 + \mathrm{CV}_1^*) + c_1} \\ \vdots \\ q_N^* = \frac{A - k \cdot q_{-N} - b_N}{k \cdot (2 + \mathrm{CV}_N^*) + c_N} \end{cases}$$
$$s.t. \quad q_{i\min} \leqslant q_i^* \leqslant q_{i\max} \qquad \sum_{i=1}^{N} q_i^* = D \qquad \pi = A - kD \tag{9}$$

where $q^* = (q_1^*, q_2^*, \cdots q_N^*)$ is the equilibrium point corresponding to the CV set $(\mathrm{CV}_1^*, \mathrm{CV}_2^*, \cdots, \mathrm{CV}_N^*)$ which is held by each GENCO when the market is cleared. It is shown that the obtained equilibrium q^* after market clearing is a Nash equilibrium even if the different GENCOs hold different CV values [18]. Different CV sets would lead to different Nash equilibriums. In the CV-based learning bidding method with the convergent CV set mentioned in the next section, the repeated market will converge to a stable Nash equilibrium.

From the next section on, the focus is on how evolutionary learning is used to search optimal CV values for each GENCO to aid the optimal bidding decision in the scenarios with and without accurate market information.

4. CV-based learning bidding in repeated market

4.1. Basic CV-based learning method with accurate market model

In the repeated market environment, the developed bidding strategies are based on available historical information. Now we investigate the dynamic CV method. CV_i^t is used to represent the CV value for GENCO i at time step t. It is assumed that the newest observation of competitors' bidding and CV value are used for decisions of the next time step, say q_i^{t-1} and CV_i^{t-1}. The coefficients of the demand curve and cost function are kept time-independent. Then, at each time step t, the bidding quantity developed in (8) becomes,

$$q_i^t = \frac{A - k \cdot q_{-i}^{t-1} - b_i}{k \cdot (2 + \mathrm{CV}_i^{t-1}) + c_i} \qquad \text{s.t.} \quad q_{i\min} \leqslant q_i^t \leqslant q_{i\max} \tag{10}$$

Now the pivotal question is how one GENCO strategically speculates the CV value in a dynamic repeated market. The simplest reasoning is that if GENCO i observes increments of the competitors' strategy e.g. bidding quantity in this case and computes the ratio with its own increments, an estimate of the proportionality factor used by its competitor can be obtained as follows [15,19]:

$$\mathrm{CV}_i^t = \frac{\Delta q_{-i}^{t-1}}{\Delta q_i^{t-2}} = \begin{cases} \frac{q_{-i}^{t-2} - q_{-i}^{t-1}}{q_i^{t-2} - q_i^{t-3}}, & |q_i^{t-2} - q_i^{t-3}| > \theta \\ \mathrm{CV}_i^{t-1}, & |q_i^{t-2} - q_i^{t-3}| \leqslant \theta \end{cases} \tag{11}$$

where $\theta > 0$ is a small predefined number to avoid a null denominator and keep the equation numerically stable.

It has already been shown in Ref. [18] that for each GENCO with an accurate uniform model of the demand curve and accurate CV estimation, the repeated game will converge to Nash equilibrium. The results in Ref. [18] showed that dynamic CV-based learning for GENCOs can help to achieve better social welfare, e.g. a lower market price. This conclusion is also validated in the simulation part of this paper in Section 5.

However, in the approaches mentioned above, the market equilibrium is achieved based on the assumption that an accurate uniform demand price function exists, which is shared by all involved GENCOs. That is in (9) the equation sets for q_i^* utilize one common demand function (i.e. the same k and A for all GENCOs). Obviously, it is hard to be supported by real world electricity markets where each GENCO has its own demand curve estimation. In the next section we will analyze the performance of the basic CV-based learning bidding algorithm when deviations exist between the estimated model of individual GENCO and the real market model.

4.2. Analyses of impact of model deviation

Various GENCOs would use different modeling methods (e.g. regression modeling) to analyze their available data and construct their own demand function. Partially available information and different exposure towards the information makes it very hard for all GENCOs in market to reach a commonly-agreed price-demand function. Practically, even with the same historical demand price pair data, various GENCOs could build slightly different demand functions.

We assume that a market demand function exists and is denoted as the reference demand function $f_r(D,t)$ which is associated with time. For each GENCO the time-dependent demand curve $f_i(D,t)$ can be denoted by the reference model $f_r(D,t)$ plus a deviation function $e_i(D,t)$ which is dependent on time as well. As we assume the estimation executed by each GENCO is comparably accurate, thus for any D and t, the absolute value of the deviation is less than a predefined value δ (>0).

$$\begin{aligned} &f_i(D,t) = f_r(D,t) + e_i(D,t) \\ &|e_i(D,t)| < \delta, \quad \forall \delta > 0 \end{aligned} \tag{12}$$

In order to better compare with the basic CV-based learning bidding introduced in the previous section, we denote,

$$f_r(D,t) = A - kD \tag{13}$$

The relation between $f_r(D,t)$ and $f_i(D,t)$ is shown in Fig. 1. The curve shown in Fig. 1 is mainly for demonstrative purposes. The actual demand curve could be much more complex than this linear form.

It is assumed that historical demand-price pair data (i.e. dots in the Fig. 1) are available by which GENCOs use to develop their regression functions for demand curves. The reference demand function $f_r(D,t)$ is indicated by the solid line in Fig. 1. $f_i(D,t), (\forall i = 1, \cdots, N)$ lies in a confidence range indicated by the area between two dotted lines in Fig. 1. A_i and k_i are the intercept and slope of the demand curve estimated by GENCO i.

When GENCO i has its own estimated demand function $f_i(D,t)$ at time t, based on (12) and (13) the derivative of its own estimated market price π_i^t with respect to q_i^t is

$$\frac{\partial \pi_i^t}{\partial q_i^t} = -k \cdot (1 + \mathrm{CV}_i^t) + \frac{\partial e_i^t}{\partial q_i^t} \tag{14}$$

When replacing (6) with (12)–(14) and rearranging, the bidding quantity at time t becomes:

$$q_i^t = \frac{A - k \cdot q_{-i}^{t-1} - b_i + e_i^t}{k \cdot (2 + \mathrm{CV}_i^{t-1}) + c_i + \frac{\partial e_i^t}{\partial q_i^t}} \qquad \text{s.t.} \quad q_{i\min} \leqslant q_i^t \leqslant q_{i\max} \tag{15}$$

Compared to (10) with the accurate uniform market model, (15) represents the bidding methods in case of the GENCOs' demand curves with deviations. It can be seen from (15) that both deviation $e_i(D,t)$ and its derivative $\frac{\partial e_i^t}{\partial q_i^t}$ influence the bidding value of GENCO. The impact of $e_i(D,t)$ on the bidding process is now discussed:

(1) If $e_i(D,t)$ is constant, say C_{e_i}, then $\frac{\partial e_i^t}{\partial q_i^t} = 0$. In this case, (15) becomes

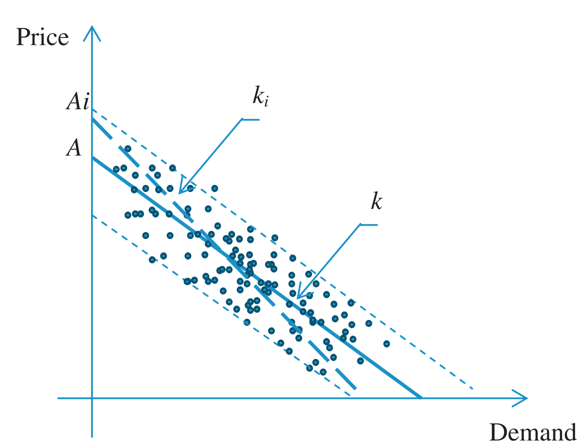

Fig. 1. Possible range of demand curves estimated by various GENCOs.

$$q_i^t = \frac{A - k \cdot q_{-i}^{t-1} - b_i + C_{e_i}}{k \cdot (2 + \mathrm{CV}_i^{t-1}) + c_i} \quad s.t. \quad q_{i\min} \leqslant q_i^t \leqslant q_{i\max} \tag{16}$$

Since C_{e_i} is constant, the item $(-b_i + C_{e_i})$ in (16) can be synthesized into one constant value, say b_i'. Then for GENCO i nothing changes compared with (10), except for pseudo coefficient b_i' replacing the real coefficient b_i of GENCO's cost function in (16). The competitors of GENCO i are able to adapt to this constant error by iterative learning.

(1) If $e_i(D, t)$ is a stochastic function $g_i(q_i^t)$ which is associated to q_i^t, then in this discrete system

$$\frac{\partial e_i^t}{\partial q_i^t} = \frac{e_i^t - e_i^{t-1}}{q_i^t - e_i^{t-1}} \neq 0 \tag{17}$$

Eq. (17) shows that the derivative $\frac{\partial e_i}{\partial q_i^t}$ is determined both by the variation of e_i and q_i in two consecutive time steps $t-1$ and t. For instance, if, in the repeated learning process, q_i gradually converges (but does not really converge) and oscillates around one value, then $(q_i^t - q_i^{t-1})$ as a small dominator makes $\frac{\partial e_i^t}{\partial q_i^t}$ very big. In this case$\frac{\partial e_i}{\partial q_i^t}$ is introduced as a non-negligible error. Furthermore, the q_i^t calculated with (16) propagates this error through the entire learning process. The simulation results in Section 5 show that errors can be accumulated and amplified during this process. With the derivative of the stochastic errors being added to each bidding time step, the game based on basic CV learning based methods cannot converge to a stable state. The reason is that the stochastic errors have no particular behavior pattern. Hence, the learning algorithm itself cannot capture this uncertainty effectively. Consequently the learnt CV value cannot reflect the real behavior variation of competitors, which cannot provide precise and effective conjectural information for the bidding decision anymore. The game becomes unstable, because each learner is unable to deal with the uncertainty if only using (11) and (10) to update the CV value and bidding quantity. In the simulation part in Section 5.1 and 5.2, it can be seen that in such case the market prices, CV values and quantities oscillate around a value.

In order to get rid of the model deviation caused by inaccurate information, we propose a CV-based bidding algorithm with information filtering technology.

4.3. CV-based learning bidding with the data filter

In this section, in order to alleviate the disadvantage resulting from the uncertainty caused by the demand estimation model deviation, an information filtering technology is introduced.

Firstly, a sliding window model is used here to discount the old data. The implication of a sliding window is that the most recent w (size of the window) CV values in the CV data stream are considered most relevant for the newest update of CV data. Furthermore in each time step the employed data stream is renewed with the data window moving onwards with one unit (e.g. one time step). Each time step, the active data in the window, i.e. the most recent w data, are used to evaluate the CV value.

Secondly, with iterative steps, the CV values are calculated in a cumulative way. With time going on, the old CV values gain more weight than the newest CV values. The details of the proposed algorithm are as follows:

Let w denote the size of window, and t denotes the time step, and then CV_i^t is inferred:

$$\mathrm{CV}_i^t = \begin{cases} \left(\frac{z-h}{z}\right) \cdot \frac{q_{-i}^{t-2} - q_{-i}^{t-1}}{q_i^{t-2} - q_i^{t-3}} + \frac{h}{z} \cdot \mathrm{CV}_i^{t-1}, & |q_i^{t-2} - q_i^{t-3}| > \theta \\ \mathrm{CV}_i^{t-1}, & |q_i^{t-2} - q_i^{t-3}| \leqslant \theta \end{cases} \tag{18}$$

If $t \geqslant w$, then $z = w$ which means that the available data stream is large enough to form a sliding window with size w.

If $t < w$, then $z = t$, when data size is not enough to form a sliding window, then all available data is processed.

h is the adjustment factor, which is able to regulate the weight between the old data and the new data. Typically, the larger h is, the slower the algorithm (18) converges, and vice versa. The errors introduced by the demand modeling deviation are alleviated during the process of weighing old and new information.

The meaningful logic behind the proposed algorithm can be interpreted from another perspective. The market information is far from complete and perfect. GENCOs might exhibit partial confidence on their new conjectural variation belief and put some reliance on the old status. That means their uncertainty about the other competitor's behavior make GENCO to adjust their bidding strategy with partial trust in the best response. Another interesting thing is that with the learning going on, the influence of small demand evaluation bias can be largely smoothed out. Correspondingly, the calculated CV values will gradually match the real market model.

5. Numerical simulations

The proposed learning algorithm is firstly studied in a duopoly system with three nodes: two GENCOs and one load for illustration purposes. The performance of the proposed algorithms in the scenarios with and without inaccurate market information will be illustrated. Afterwards, a complex system with more market players, i.e. 6 GENCOs and one load will be given to show the bidding process in a more competitive market.

5.1. Cases with 2 players

The cost function data of 2 GENCOs are shown in Table 1. The reference system demand function has a form as in (5), whose details are shown in Table 2.

In all four next cases, the simulation runs for 250 rounds. It is supposed that all GENCOs in the simulations are able to learn. The initial CV values for four cases are set equally, say -0.8 for both GENCOs. The initial bidding quantities of both GENCOs are pre-assigned as 446.99 (MWh) and 709.98 (MWh) for four cases.

The focus is put on validation of the analyses about the modeling deviation described in Section 4 and on comparing the algorithm with and without data filters. The equilibrium and market outcome are analyzed as well.

5.1.1. Case A: Basic CV-based bidding in an accurate market information environment for 2-player system

In this case, the performance of the basic CV-based bidding algorithm (CVBA) is examined and set as the benchmark case. One uniform and accurate market model is shared by two GENCOs. The simulation results are shown in Figs. 2a and 2b.

As can be seen from Fig. 2a, after 4 rounds with the necessary accumulated historical data, both GENCOs begin to learn from historical data. Then, we can see the fluctuation of both CV values and bidding quantities as both GENCOs try to estimate their competitor's behavior. After about 11 rounds, the system reaches a stable state. The stable CV for Gen 1 is -0.39 and -0.33 for Gen 2. The

Table 1
Generator data.

	b	c	Min capacity (MWh)	Max capacity (MWh)
Gen1	3	0.025	0	800
Gen2	1.75	0.0175	0	800

Table 2
Reference demand function data.

	Max capacity (MWh)	A	k
Reference demand	1500	35	0.018

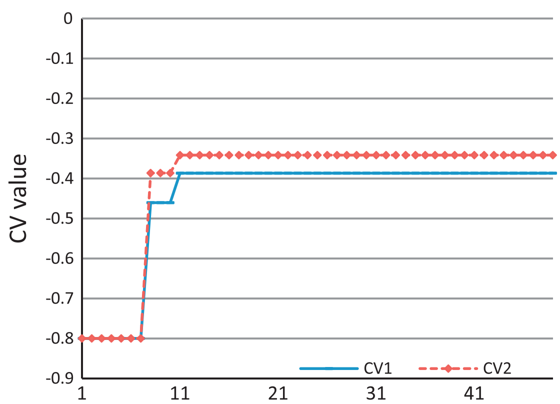

Fig. 2a. CV values in Case A.

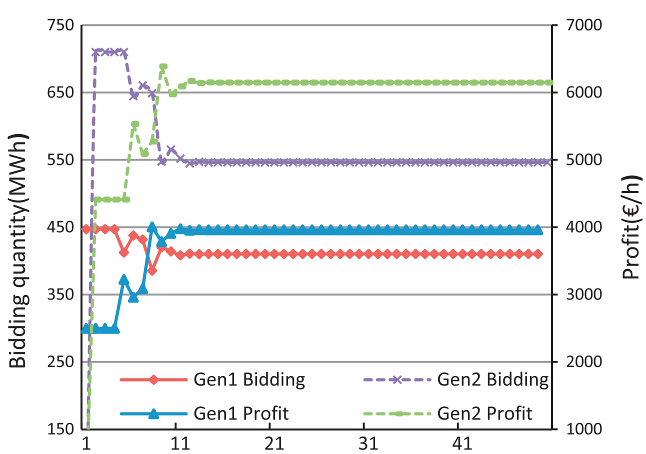

Fig. 2b. Bidding quantities and profits for Gen1 and Gen2 in Case A.

stable bidding quantity for Gen 1 is 410 (MWh) and 546 (MWh) for Gen 2. As proved in Ref. [18], this state is a Nash equilibrium. The evolved strategy after the learning converges is Nash strategy. Any deviation from the convergent bidding quantity and the related CV value for any GENCO will cause loss of profit. The profits for Gen 1 and Gen 2 are 3961 (€/h) and 6148 (€/h) respectively. The market price after convergence is 17.80 (€/MWh).

Moreover, considering the fact that other GENCOs adjust their behavior simultaneously, the results show that the basic CVBA in this duopoly game reaches the equilibrium state rather fast. It can be interpreted as follows: if we describe the CV-based bidding process as a process in which a GENCO searches the optimal solution, given a state space consisting of the factors e.g. market states and other players' behaviors, then the duopoly game with the accurate demand function means that the number of states in this process is less than the multiple player system in certain demand environment.

5.1.2. Case B-1: Basic CV-based bidding in constant model deviation environment for 2-player system

In this case, we investigate the impact of model deviations on the market demand function held by different GENCOs. Firstly, we study the constant deviation which does not change over time. The model deviation from the reference demand function is arbitrarily set to 1.5 and −1.5 for Gen1 and Gen2 respectively. The simulation results of the market bidding process are shown in Figs. 3a and 3b.

It can be seen from the figure that the market reaches an equilibrium state even when model deviations exist among GENCOs. The convergence of the bidding behaviors and CV values for both GENCOs is quite similar to Case A. The learning process converges very fast. After about 14 rounds, the system reaches a stable state. The stable CV for Gen 1 is −0.38 and −0.34 for Gen 2. These CV values are quite close to those in Case A. Fig. 3b shows that the profits for Gen 1 and Gen 2 are 4174 (€/h) and 5863 (€/h) respectively when the system enter a stable state. The market price after convergence is 17.87 (€/MW h). These values have small deviations from those in Case A. This deviation is introduced by the constant model deviations held by two GENCOs.

The simulation results verify the analyses in Section 4 for the scenario where the model deviations are constant. Each GENCO implicitly takes this deviation into its bidding evaluation and adapts to the constant deviation via the repeated learning.

5.1.3. Case B-2: Basic CV-based bidding in stochastic model deviation environment for 2-player system

In this case, it is supposed that the various GENCOs interpret the historical demand data in different ways and the induced errors are related with the bidding quantities in a stochastic manner. Small stochastic noise is added to the reference demand function data. Then the two GENCOs perceive the new demand functions as follows:

$$f_i(D,t) = 35 - 0.018 \cdot D + 0.25 - 0.5 \cdot P(0,1)$$

where $P(0, 1)$ is a standard uniform distribution. The related market simulation is shown in Figs. 4a and 4b.

We can see from Fig. 4a that even with these small deviations (−0.25–0.25 compared to $A = 35$), the CV values of each GENCO do not converge to a certain value, but oscillate. Instead, it changes significantly even in two consecutive bidding time steps. Consequently, bidding quantity of each GENCO cannot converge to a certain value either (see Fig. 4b). This significant change between two adjacent time steps makes the system quite unstable. This means that the basic CVBA cannot work efficiently in the uncertainty environment where model deviations exist.

The source of these big changes is that the model deviations influence the computation of CV values which in turn changes the bidding quantities of GENCOs. Even small model deviations will be amplified during the learning process. In next section, the

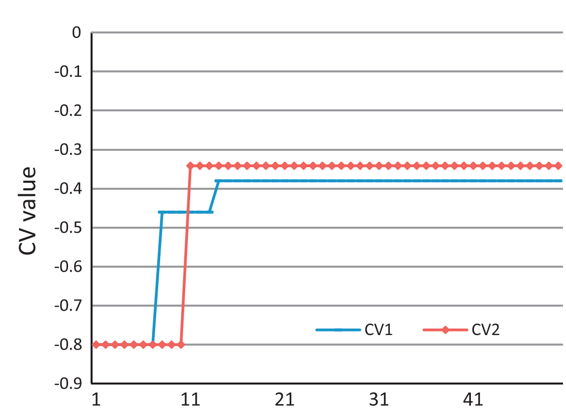

Fig. 3a. CV values in Case B-1.

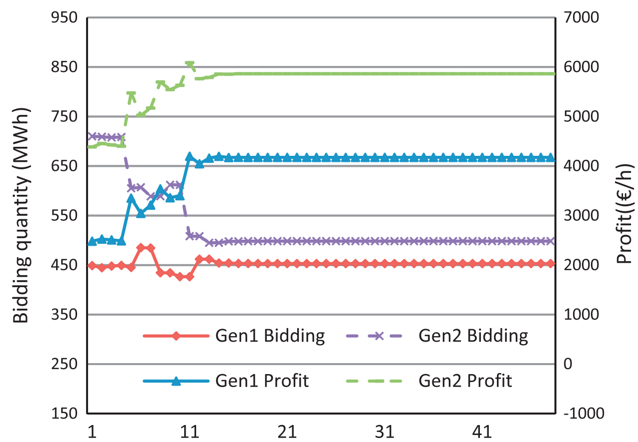

Fig. 3b. Bidding quantities and profits for Gen1 and Gen2 in Case B-1.

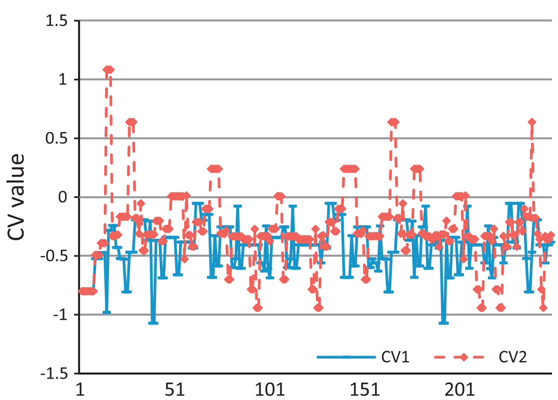

Fig. 4a. CV values in Case B-2.

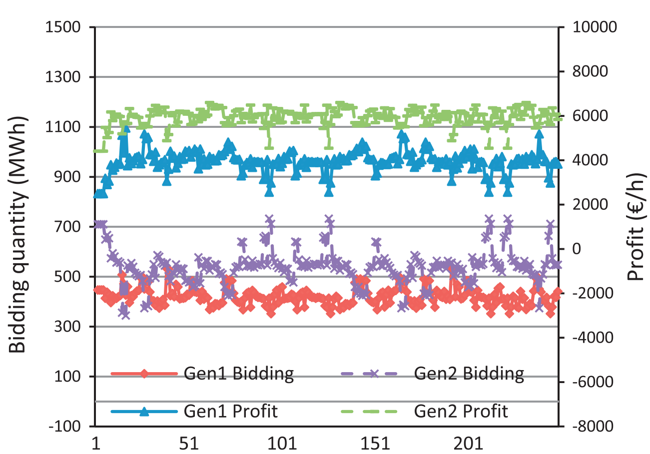

Fig. 4b. Bidding quantities and profits for Gen1 and Gen2 in Case B-2.

bidding algorithm with data filters will be examined, to cope with this problem.

5.1.4. Case C: CV-based bidding with data filters in stochastic model deviation environment for 2-player system

In this case, the CV-based learning bidding with data filtering is employed for the same noise setting as in Case B-2. The length of sliding window, w, is set to be 50. The adjustment factor h is taken to be 3. The related market information is shown in Figs. 5a and 5b.

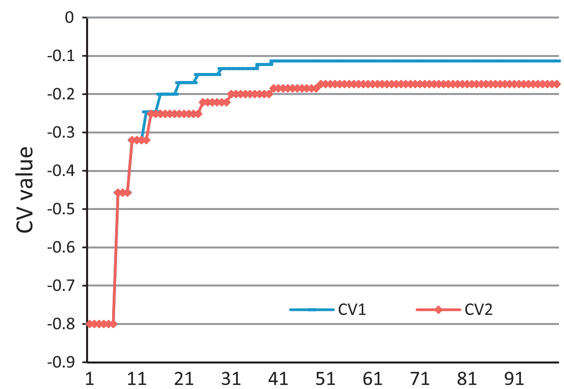

Fig. 5a. CV values in Case C.

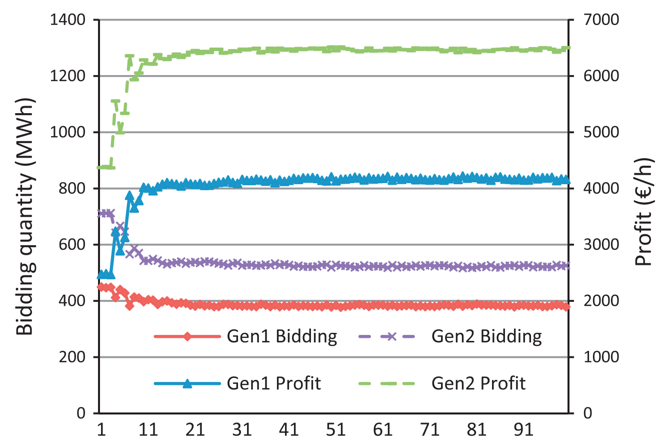

Fig. 5b. Bidding quantities and profits for Gen1 and Gen2 in Case C.

As can be seen in Fig. 5a, the trend of the bidding curve becomes more flat and stable compared with Case B-2. After around 50 rounds, the learning algorithms tend to come to a stable state with minor oscillations as the model deviations remain always present. This quite differs from the simulation results shown in Case B-2 in which the big oscillations can be easily identified. Moreover, the convergence value in this case differs from the one in Case A and B-1 because of the existence of the data filter.

It shows that the stable CV value for Gen 1 is −0.11 and −0.17 for Gen 2. The stable profit for Gen 1 is around 4172 (€/h) and around 6464 (€/h) for Gen2. The convergent market price is 18.68 (€/MWh). Both the CV value and the market price become higher than the equilibrium state in benchmark Case A. This change is due to the fact that the data filtering removes not only the noise from model deviations but also the useful information of other GENCOs' reaction in the learning process. Then it leads a GENCO to believe that the market competition level is not very high, which is reflected by the higher CV estimation values and it influences the bidding decision in turn.

Table 3
Generator data for 6 GENCOs.

	b	c	Min capacity (MWh)	Max capacity (MWh)
GENCO 1	2	0.025	0	480
GENCO 2	1.75	0.0175	0	480
GENCO 3	3	0.025	0	300
GENCO 4	3	0.025	0	300
GENCO 5	1	0.0625	0	300
GENCO 6	3.25	0.00834	0	480

Table 4
Reference demand function data in 6 GENCO system.

	Max capacity (MWh)	*A*	*k*
Reference demand	2500	50	0.02

5.2. Case with 6 players

In this section, the proposed algorithms are tested in a more competitive market, i.e. 6 GENCOs system. In such multi-player market, the competition is tougher than that in the duopoly case studied in Section 5.1. As a result of the increased number of players a GENCO faces more varying behavior changes. The cost function data of the GENCOs and the reference demand function are shown in Tables 3 and 4 respectively. Two cases with basic CV bidding strategy and CV-based bidding with filter are shown in the subsection 5.2.1 and 5.2.2.

5.2.1. Case D: Basic CV-based bidding in stochastic model deviation environment for 6-player system

The same stochastic noise used in Case B-2 is adopted. However the amplitude of noise added to the demand function in this case is smaller compared to that in Section 5.1 since the demand curve has a larger intersection. The simulated market information is shown in Figs. 6a and 6b.

It can be seen in Fig. 6a that in this stochastic noise case market cannot reach a stable state even after 150 bidding rounds. Partially due to the demand errors, the CV value of each GENCO oscillates between several values. The facts that all CV values are below zero validate the hypothesis that the competition in the multi-player market is sharper compared to the duopoly case. Moreover each GENCO in this case has smaller market shares. These two characteristics make the instable situation less chaotic and explains why the bidding quantities of GENCOs in Fig. 6b evolve following a particular pattern (if small noises are ignored). In this multi-player case, the bidding quantities of the GENCOs in some time slots reach their maximal capacities. That shows that GENCOs always seek to output as much as possible.

5.2.2. Case E: CV-based bidding with data filters in stochastic model deviation environment for 6-player system

In this case, the CV-based learning bidding with the data filtering is employed. The length of the sliding window, w, is set to 10. The adjustment factor h is taken to be 1. The considerations are that more uncertainties require players, on one hand, to be flexible enough to adapt to the new system change but, on the other hand, still use reservation on this new information. The simulation results are shown in Figs. 7a and 7b.

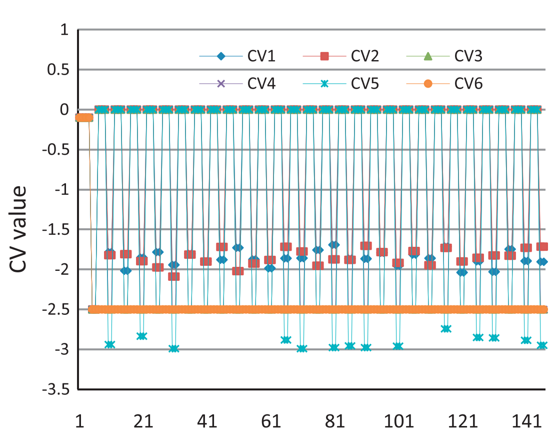

Fig. 6a. CV values in Case D.

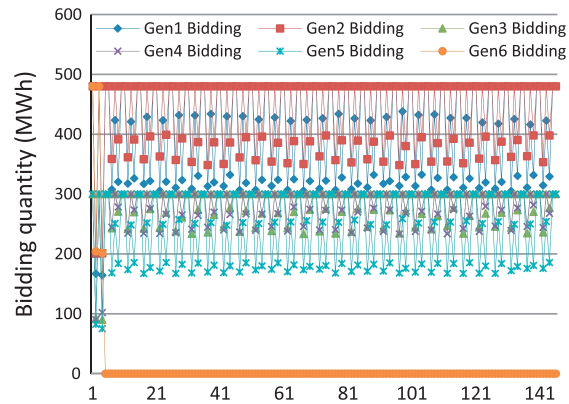

Fig. 6b. Bidding quantities for 6 Gens in Case D.

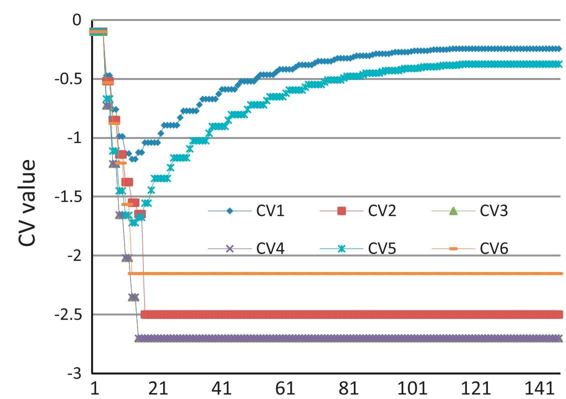

Fig. 7a. CV values in Case E.

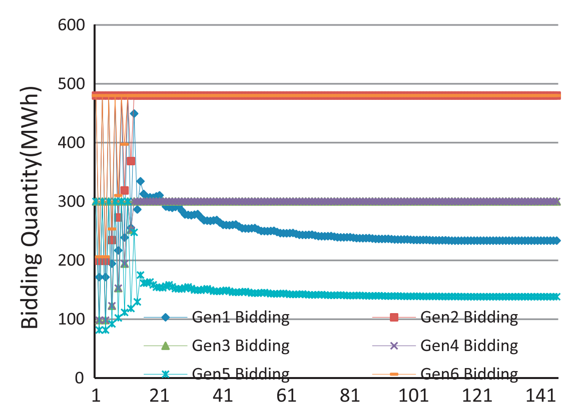

Fig. 7b. Bidding quantities for 6 Gens in Case E.

Fig. 7a shows that the proposed method alleviates the instability indicated in Case D. Fig. 7b shows that the system converges after more than 70 rounds. The interesting thing from Fig. 7b is that the convergent values for several GENCOs (Gen 2, 3, 4 and 6) are their maximal capacity. The reason is that they are the cheapest producers. With the same logic, the most expensive GENCO, i.e. Gen 5, outputs least. This evidence shows that, even if GENCOs have strategic bidding abilities, their bidding behavior is still constrained by their own characteristics. The CV values of all GENCOs in Fig. 7a are negative. It validates that more players imply more competition, which matches the economic definition of a competitive market.

6. Conclusion

This paper starts from a practical concern: due to the complexity of the market and the lack of complete market information, various GENCOs formulate different market models even if based on the same historical market data. As a consequence, such model deviations can impact the GENCOs' bidding behavior as well as the market performance. It is shown that the basic CV-based bidding process cannot work effectively and might bring the system into an unstable state when the market model variations exist. Analyses have been done to illustrate the impact of such variation on the bidding decision process. A set of simulations verified the conclusions from the theoretical analyses. A CV-based dynamic learning algorithm with data filtering is proposed to alleviate the influence of the diversities of the evaluated demand model. The simulation results show that the proposed algorithms achieve better performance of convergence and stability compared with the basic CV-based bidding without data filtering. Although this paper limits the effects of the model deviations to equilibrium-oriented bidding method, the analyses developed in this paper can provide an important guidance to other bidding methods.

The model deviation discussed here is a simple one (i.e. linear form), hence the proposed algorithm, with simple filtering functions, achieves good performance. However, this algorithm can be further improved. As discussed in the simulation of Case C, this algorithm filters out not only noise but also useful information representing GENCOs' behavioral reactions. Further work can be done to analyze more complex model deviation forms and to develop a more efficient bidding algorithm with a proper filter that can filter out model noise while keeping useful information. Another perspective of the future work is to extend the uniform pricing model used in this paper to a system with the locational market price (LMP) where the effect of congestion of transmission lines has to be taken into account, which introduces another source of model deviation.

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